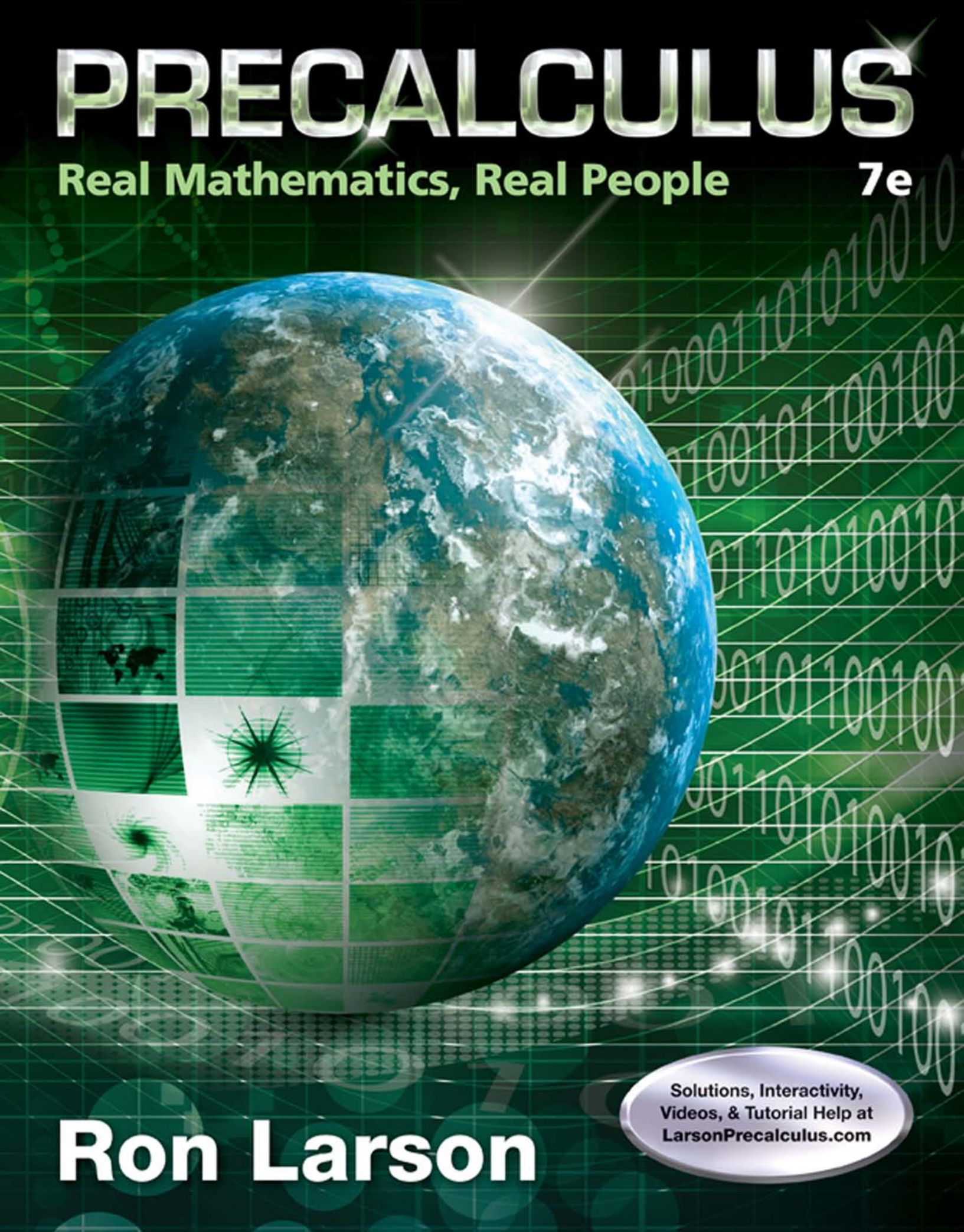
PRECALCULUS

Real Mathematics, Real People 7e



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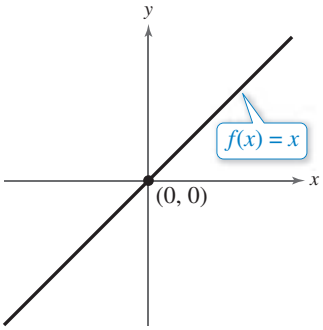
Ron Larson

Library of Parent Functions Summary

Linear Function *(p. 6)*

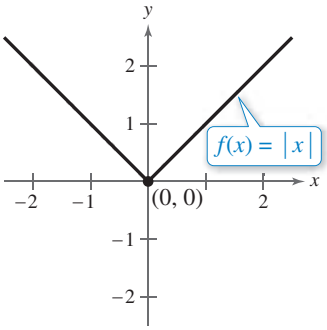
$f(x) = x$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(0, 0)$
Increasing

Absolute Value Function *(p. 19)*

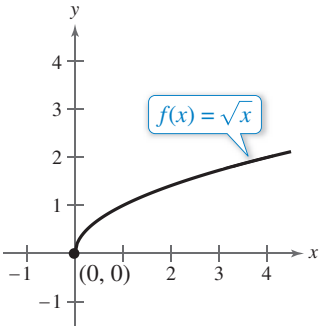
$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$



Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$
Even function
y-axis symmetry

Square Root Function *(p. 20)*

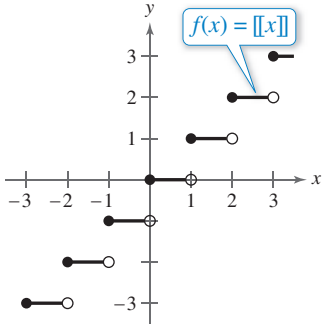
$f(x) = \sqrt{x}$



Domain: $[0, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Increasing on $(0, \infty)$

Greatest Integer Function *(p. 34)*

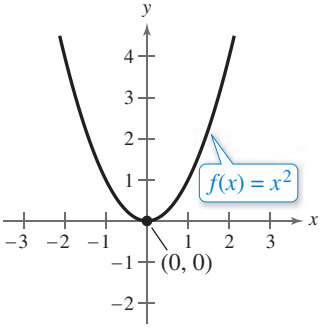
$f(x) = [\![x]\!]$



Domain: $(-\infty, \infty)$
Range: the set of integers
x-intercepts: in the interval $[0, 1)$
y-intercept: $(0, 0)$
Constant between each pair of consecutive integers
Jumps vertically one unit at each integer value

Quadratic Function *(p. 92)*

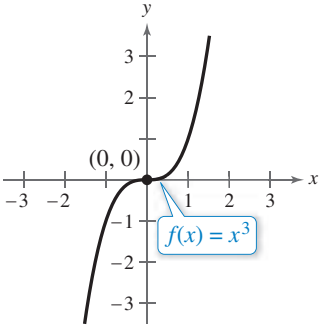
$f(x) = x^2$



Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$
Even function
Axis of symmetry: $x = 0$
Relative minimum or vertex: $(0, 0)$

Cubic Function *(p. 101)*

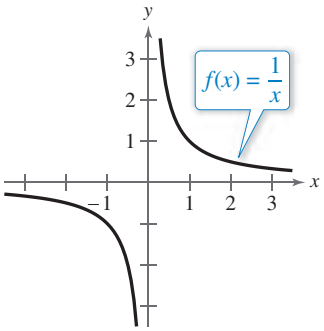
$f(x) = x^3$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(0, 0)$
Increasing on $(-\infty, \infty)$
Odd function
Origin symmetry

Rational Function ***(p. 152)***

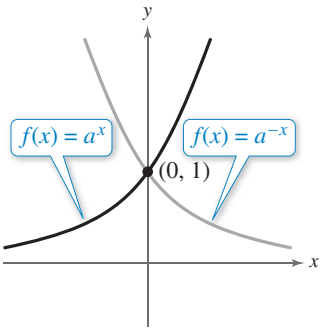
$f(x) = \dfrac{1}{x}$



Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$
No intercepts
Decreasing on $(-\infty, 0)$ and $(0, \infty)$
Odd function
Origin symmetry
Vertical asymptote: y-axis
Horizontal asymptote: x-axis

Exponential Function ***(p. 182)***

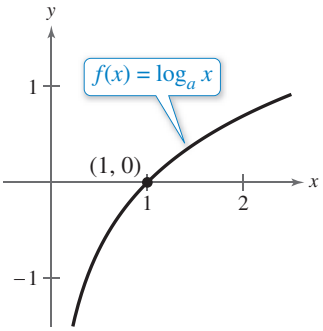
$f(x) = a^x, a > 1$



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Intercept: $(0, 1)$
Increasing on $(-\infty, \infty)$
for $f(x) = a^x$
Decreasing on $(-\infty, \infty)$
for $f(x) = a^{-x}$
x-axis is a horizontal asymptote
Continuous

Logarithmic Function ***(p. 195)***

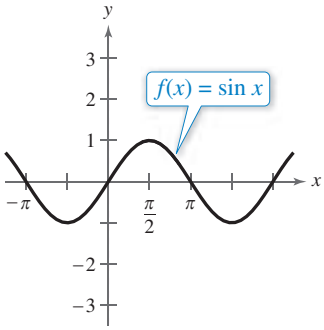
$f(x) = \log_a x, a > 1$



Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(1, 0)$
Increasing on $(0, \infty)$
y-axis is a vertical asymptote
Continuous
Reflection of graph of $f(x) = a^x$
in the line $y = x$

Sine Function ***(p. 293)***

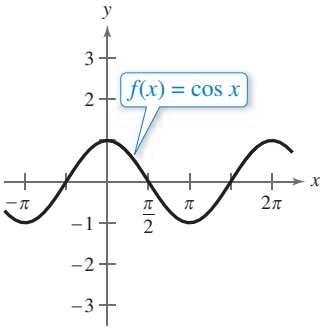
$f(x) = \sin x$



Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π
x-intercepts: $(n\pi, 0)$
y-intercept: $(0, 0)$
Odd function
Origin symmetry

Cosine Function ***(p. 293)***

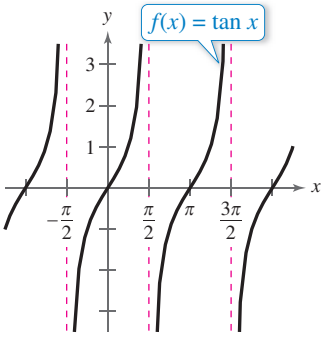
$f(x) = \cos x$



Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π
x-intercepts: $\left(\dfrac{\pi}{2} + n\pi, 0\right)$
y-intercept: $(0, 1)$
Even function
y-axis symmetry

Tangent Function ***(p. 304)***

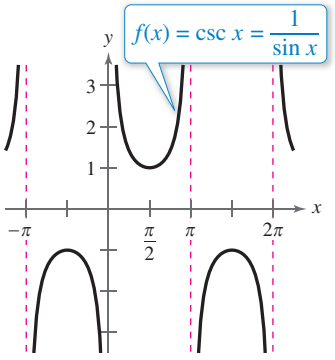
$f(x) = \tan x$



Domain: $x \neq \dfrac{\pi}{2} + n\pi$
Range: $(-\infty, \infty)$
Period: π
x-intercepts: $(n\pi, 0)$
y-intercept: $(0, 0)$
Vertical asymptotes: $x = \dfrac{\pi}{2} + n\pi$
Odd function
Origin symmetry

Cosecant Function *(p. 307)*

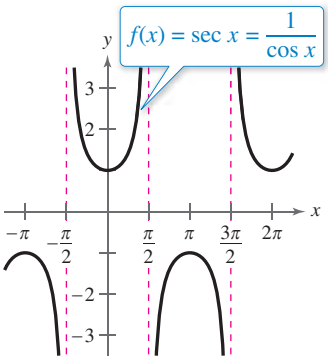
$f(x) = \csc x$



Domain: $x \neq n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π
No intercepts
Vertical asymptotes: $x = n\pi$
Odd function
Origin symmetry

Secant Function *(p. 307)*

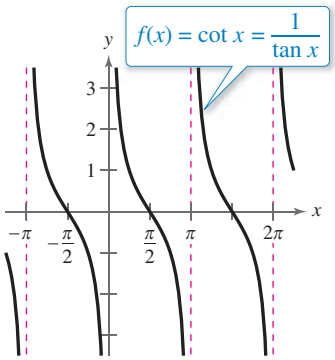
$f(x) = \sec x$



Domain: $x \neq \frac{\pi}{2} + n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π
y-intercept: $(0, 1)$
Vertical asymptotes:
$x = \frac{\pi}{2} + n\pi$
Even function
y-axis symmetry

Cotangent Function *(p. 306)*

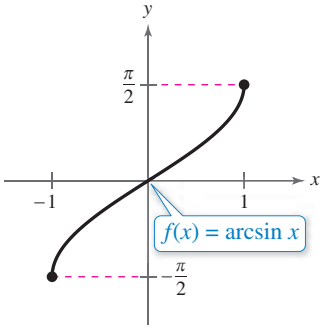
$f(x) = \cot x$



Domain: $x \neq n\pi$
Range: $(-\infty, \infty)$
Period: π
x-intercepts: $\left(\frac{\pi}{2} + n\pi, 0\right)$
Vertical asymptotes: $x = n\pi$
Odd function
Origin symmetry

Inverse Sine Function *(p. 319)*

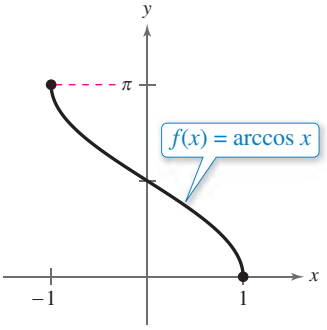
$f(x) = \arcsin x$



Domain: $[-1, 1]$
Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Intercept: $(0, 0)$
Odd function
Origin symmetry

Inverse Cosine Function *(p. 319)*

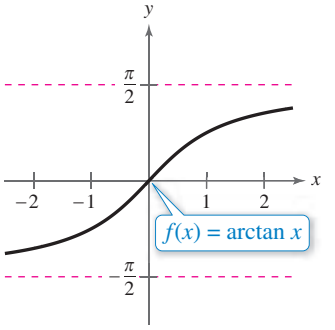
$f(x) = \arccos x$



Domain: $[-1, 1]$
Range: $[0, \pi]$
y-intercept: $\left(0, \frac{\pi}{2}\right)$

Inverse Tangent Function *(p. 319)*

$f(x) = \arctan x$



Domain: $(-\infty, \infty)$
Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
Intercept: $(0, 0)$
Horizontal asymptotes: $y = \pm\frac{\pi}{2}$
Odd function
Origin symmetry

Precalculus
Real Mathematics, Real People
Seventh Edition

Ron Larson
The Pennsylvania State University
The Behrend College

With the assistance of David C. Falvo
The Pennsylvania State University
The Behrend College

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Seventh Edition

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Preface

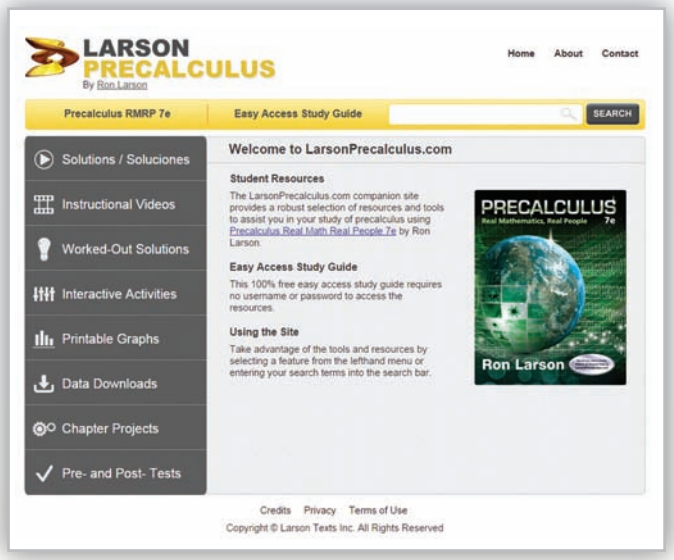
Welcome to *Precalculus: Real Mathematics, Real People*, Seventh Edition. I am proud to present to you this new edition. As with all editions, I have been able to incorporate many useful comments from you, our user. And while much has changed in this revision, you will still find what you expect—a pedagogically sound, mathematically precise, and comprehensive textbook. While we need the mathematics of precalculus to master calculus, precalculus is not simply preliminary material for calculus. It stands alone as "real mathematics" in itself. In this book you will see how precalculus is used by real people to solve real-life problems and make real-life decisions.

In addition to providing real and relevant mathematics, I am pleased and excited to offer you something brand new—a companion website at **LarsonPrecalculus.com.** My goal is to provide students with the tools they need to master precalculus.

New To This Edition

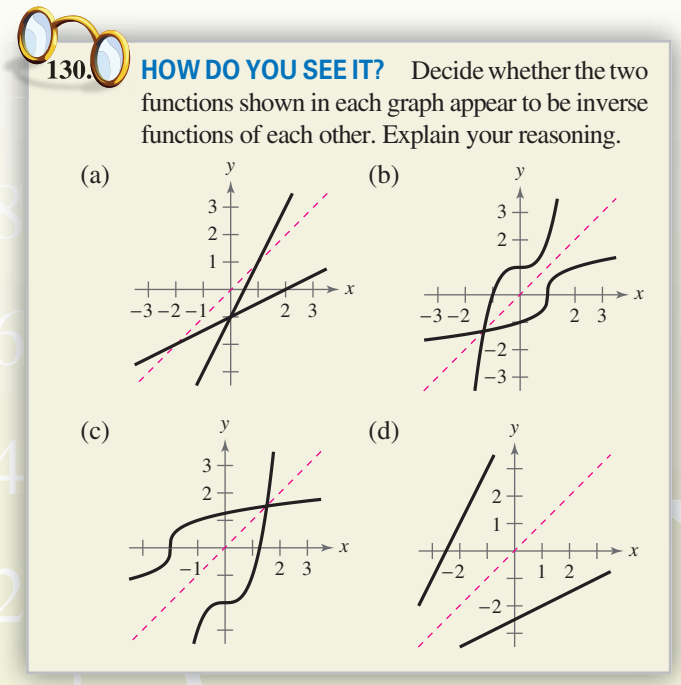
NEW LarsonPrecalculus.com

This companion website offers multiple tools and resources to supplement your learning. Access to these features is free. View and listen to worked-out solutions of Checkpoint problems in English or Spanish, explore examples, download data sets, watch lesson videos, and much more.



NEW Checkpoints

Accompanying every example, the Checkpoint problems encourage immediate practice and check your understanding of the concepts presented in the example. View and listen to worked-out solutions of the Checkpoint problems in English or Spanish at LarsonPrecalculus.com.


130. **HOW DO YOU SEE IT?** Decide whether the two functions shown in each graph appear to be inverse functions of each other. Explain your reasoning.

(a)

(b)

(c)

(d)

NEW How Do You See It?

The How Do You See It? feature in each section presents an exercise that you will solve by visual inspection using the concepts learned in the lesson. This exercise is excellent for classroom discussion or test preparation.

NEW Data Spreadsheets

Download these editable spreadsheets from LarsonPrecalculus.com and use the data to solve exercises.

REVISED Exercise Sets

The exercise sets have been carefully and extensively examined to ensure they are rigorous and relevant and to include all topics our users have suggested. The exercises have been **reorganized and titled** so you can better see the connections between examples and exercises. Multi-step exercises reinforce problem-solving skills and mastery of concepts by giving you the opportunity to apply the concepts in real-life situations.

REVISED Remarks

These hints and tips reinforce or expand upon concepts, help you learn how to study mathematics, address special cases, or show alternative or additional steps to a solution of an example.

Trusted Features

Calc Chat

For the past several years, an independent website—CalcChat.com—has provided free solutions to all odd-numbered problems in the text. Thousands of students have visited the site for practice and help with their homework.

Side-By-Side Examples

Throughout the text, we present solutions to examples from multiple perspectives—algebraically, graphically, and numerically. The side-by-side format of this pedagogical feature helps you to see that a problem can be solved in more than one way and to see that different methods yield the same result. The side-by-side format also addresses different learning styles.

Why You Should Learn It Exercise

An engaging real-life application of the concepts in the section. This application exercise is typically described in the section opener as a motivator for the section.

Library of Parent Functions

To facilitate familiarity with the basic functions, several elementary and nonelementary functions have been compiled as a Library of Parent Functions. Each function is introduced at its first appearance in the text with a definition and description of basic characteristics. The Library of Parent Functions Examples are identified in the title of the example and there is a Review of Library of Parent Functions after Chapter 4. A summary of functions is presented on the inside cover of this text.

Make a Decision Exercises

The Make a Decision exercises at the end of selected sections involve in-depth applied exercises in which you will work with large, real-life data sets, often creating or analyzing models. These exercises are offered online at LarsonPrecalculus.com.

Chapter Openers

Each Chapter Opener highlights a real-life modeling problem, showing a graph of the data, a section reference, and a short description of the data.

Explore the Concept

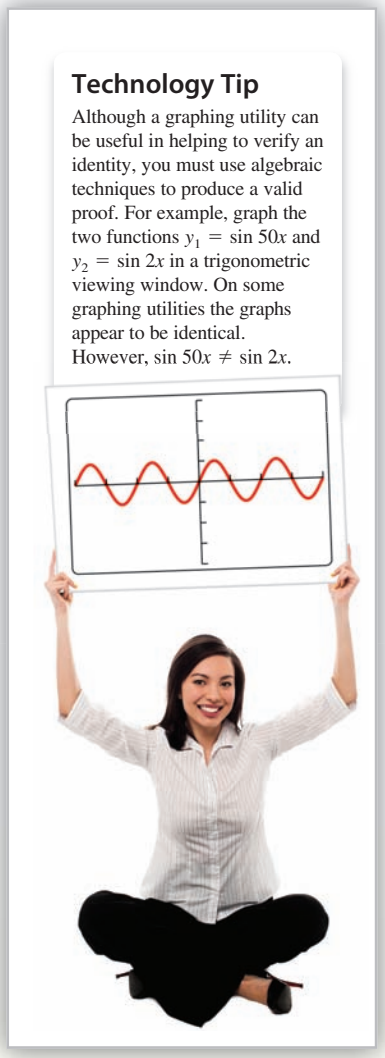
Each Explore the Concept engages you in active discovery of mathematical concepts, strengthens critical thinking skills, and helps build intuition.

Explore the Concept

Complete the following:

$i^1 = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$
$i^5 =$ ☐
$i^6 =$ ☐
$i^7 =$ ☐
$i^8 =$ ☐
$i^9 =$ ☐
$i^{10} =$ ☐
$i^{11} =$ ☐
$i^{12} =$ ☐

What pattern do you see? Write a brief description of how you would find i raised to any positive integer power.



What's Wrong?

Each What's Wrong? points out common errors made using graphing utilities.

Technology Tip

Technology Tips provide graphing calculator tips or provide alternative methods of solving a problem using a graphing utility.

Algebra of Calculus

Throughout the text, special emphasis is given to the algebraic techniques used in calculus. Algebra of Calculus examples and exercises are integrated throughout the text and are identified by the symbol ∫.

Algebraic-Graphical-Numerical Exercises

These exercises allow you to solve a problem using multiple approaches—algebraic, graphical, and numerical. This helps you to see that a problem can be solved in more than one way and to see that different methods yield the same result.

Modeling Data Exercises

These multi-part applications that involve real-life data offer you the opportunity to generate and analyze mathematical models.

Vocabulary and Concept Check

The Vocabulary and Concept Check appears at the beginning of the exercise set for each section. Each of these checks asks fill-in-the-blank, matching, and non-computational questions designed to help you learn mathematical terminology and to test basic understanding of that section's concepts.

What you should learn/Why you should learn it

These summarize important topics in the section and why they are important in math and in life.

Chapter Summaries

The Chapter Summary includes explanations and examples of the objectives taught in the chapter.

Error Analysis Exercises

This exercise presents a sample solution that contains a common error which you are asked to identify.

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Acknowledgments

I would like to thank my colleagues who have helped me develop this program. Their encouragement, criticisms, and suggestions have been invaluable to me.

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I hope that you enjoy learning the mathematics presented in this text. More than that, I hope you gain a new appreciation for the relevance of mathematics to careers in science, technology, business, and medicine.

My thanks to Robert Hostetler, The Behrend College, The Pennsylvania State University, Bruce Edwards, University of Florida, and David Heyd, The Behrend College, The Pennsylvania State University, for their significant contributions to previous editions of this text.

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If you have suggestions for improving this text, please feel free to write me. Over the past two decades I have received many useful comments from both instructors and students, and I value these very much.

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Penn State University
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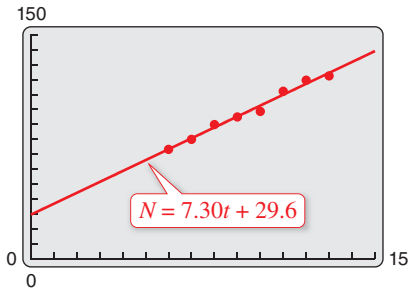
1 Functions and Their Graphs



1.1 Lines in the Plane
1.2 Functions
1.3 Graphs of Functions
1.4 Shifting, Reflecting, and Stretching Graphs
1.5 Combinations of Functions
1.6 Inverse Functions
1.7 Linear Models and Scatter Plots

Section 1.7, Example 4
e-Filed Tax Returns

Andres Rodriguez | Dreamstime

Introduction to Library of Parent Functions

In Chapter 1, you will be introduced to the concept of a *function*. As you proceed through the text, you will see that functions play a primary role in modeling real-life situations.

There are three basic types of functions that have proven to be the most important in modeling real-life situations. These functions are algebraic functions, exponential and logarithmic functions, and trigonometric and inverse trigonometric functions. These three types of functions are referred to as the *elementary functions*, though they are often placed in the two categories of *algebraic functions* and *transcendental functions*. Each time a new type of function is studied in detail in this text, it will be highlighted in a box similar to those shown below. The graphs of these functions are shown on the inside covers of this text.

Algebraic Functions

These functions are formed by applying algebraic operations to the linear function $f(x) = x$.

Name	*Function*	*Location*
Linear	$f(x) = x$	Section 1.1
Quadratic	$f(x) = x^2$	Section 2.1
Cubic	$f(x) = x^3$	Section 2.2
Rational	$f(x) = \frac{1}{x}$	Section 2.7
Square root	$f(x) = \sqrt{x}$	Section 1.2

Transcendental Functions

These functions cannot be formed from the linear function by using algebraic operations.

Name	*Function*	*Location*
Exponential	$f(x) = a^x, a > 0, a \neq 1$	Section 3.1
Logarithmic	$f(x) = \log_a x, x > 0, a > 0, a \neq 1$	Section 3.2
Trigonometric	$f(x) = \sin x$	Section 4.5
	$f(x) = \cos x$	Section 4.5
	$f(x) = \tan x$	Section 4.6
	$f(x) = \csc x$	Section 4.6
	$f(x) = \sec x$	Section 4.6
	$f(x) = \cot x$	Section 4.6
Inverse trigonometric	$f(x) = \arcsin x$	Section 4.7
	$f(x) = \arccos x$	Section 4.7
	$f(x) = \arctan x$	Section 4.7

Nonelementary Functions

Some useful nonelementary functions include the following.

Name	*Function*	*Location*
Absolute value	$f(x) = \lvert x \rvert$	Section 1.2
Greatest integer	$f(x) = [\![x]\!]$	Section 1.3

1.1 Lines in the Plane

The Slope of a Line

In this section, you will study lines and their equations. The **slope** of a nonvertical line represents the number of units the line rises or falls vertically for each unit of horizontal change from left to right. For instance, consider the two points

(x_1, y_1) and (x_2, y_2)

on the line shown in Figure 1.1.

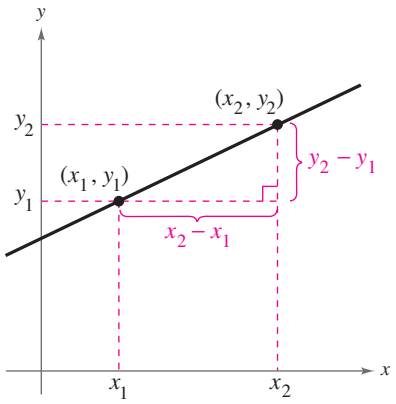


Figure 1.1

As you move from left to right along this line, a change of $(y_2 - y_1)$ units in the vertical direction corresponds to a change of $(x_2 - x_1)$ units in the horizontal direction. That is,

$y_2 - y_1 =$ the change in y

and

$x_2 - x_1 =$ the change in x.

The slope of the line is given by the ratio of these two changes.

Definition of the Slope of a Line

The **slope** m of the nonvertical line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$$

where $x_1 \neq x_2$.

When this formula for slope is used, the *order of subtraction* is important. Given two points on a line, you are free to label either one of them as (x_1, y_1) and the other as (x_2, y_2). Once you have done this, however, you must form the numerator and denominator using the same order of subtraction.

$$\underbrace{m = \frac{y_2 - y_1}{x_2 - x_1}}_{\text{Correct}} \qquad \underbrace{m = \frac{y_1 - y_2}{x_1 - x_2}}_{\text{Correct}} \qquad \underbrace{m = \frac{y_2 - y_1}{x_1 - x_2}}_{\text{Incorrect}}$$

Throughout this text, the term *line* always means a *straight* line.

What you should learn

- Find the slopes of lines.
- Write linear equations given points on lines and their slopes.
- Use slope-intercept forms of linear equations to sketch lines.
- Use slope to identify parallel and perpendicular lines.

Why you should learn it

The slope of a line can be used to solve real-life problems. For instance, in Exercise 97 on page 14, you will use a linear equation to model student enrollment at Penn State University.

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EXAMPLE 1 Finding the Slope of a Line

Find the slope of the line passing through each pair of points.

a. $(-2, 0)$ and $(3, 1)$ **b.** $(-1, 2)$ and $(2, 2)$ **c.** $(0, 4)$ and $(1, -1)$

Solution

Difference in y-values

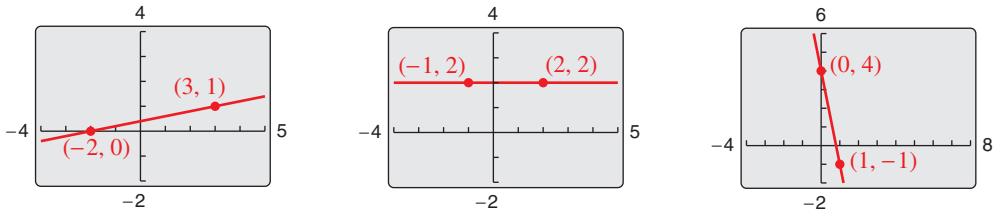
a. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1 - 0}{3 - (-2)} = \dfrac{1}{3 + 2} = \dfrac{1}{5}$

Difference in x-values

b. $m = \dfrac{2 - 2}{2 - (-1)} = \dfrac{0}{3} = 0$

c. $m = \dfrac{-1 - 4}{1 - 0} = \dfrac{-5}{1} = -5$

The graphs of the three lines are shown in Figure 1.2. Note that the *square setting* gives the correct "steepness" of the lines.



(a) **(b)** **(c)**

Figure 1.2

Explore the Concept

Use a graphing utility to compare the slopes of the lines $y = 0.5x$, $y = x$, $y = 2x$, and $y = 4x$. What do you observe about these lines? Compare the slopes of the lines $y = -0.5x$, $y = -x$, $y = -2x$, and $y = -4x$. What do you observe about these lines? (*Hint:* Use a *square setting* to obtain a true geometric perspective.)

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Find the slope of the line passing through each pair of points.

a. $(-5, -6)$ and $(2, 8)$ **b.** $(4, 2)$ and $(2, 5)$ **c.** $(0, -1)$ and $(3, -1)$

The definition of slope does not apply to vertical lines. For instance, consider the points $(3, 4)$ and $(3, 1)$ on the vertical line shown in Figure 1.3. Applying the formula for slope, you obtain

$$m = \frac{4 - 1}{3 - 3} = \frac{3}{0}.$$ Undefined

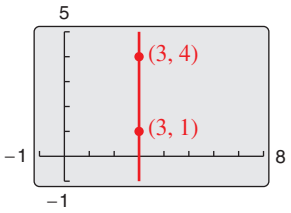

Figure 1.3

Because division by zero is undefined, the slope of a vertical line is undefined.

From the lines shown in Figures 1.2 and 1.3, you can make the following generalizations about the slope of a line.

The Slope of a Line

1. A line with positive slope ($m > 0$) *rises* from left to right.
2. A line with negative slope ($m < 0$) *falls* from left to right.
3. A line with zero slope ($m = 0$) is *horizontal*.
4. A line with undefined slope is *vertical*.

The Point-Slope Form of the Equation of a Line

When you know the slope of a line *and* you also know the coordinates of one point on the line, you can find an equation of the line. For instance, in Figure 1.4, let (x_1, y_1) be a point on the line whose slope is m. When (x, y) is any *other* point on the line, it follows that

$$\frac{y - y_1}{x - x_1} = m.$$

This equation in the variables x and y can be rewritten in the **point-slope form** of the equation of a line.

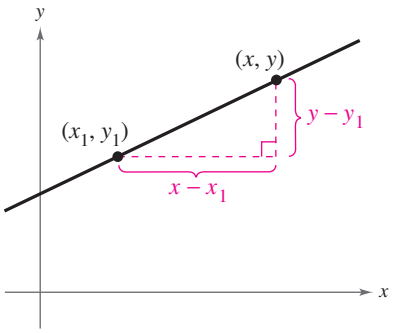

Figure 1.4

Point-Slope Form of the Equation of a Line

The **point-slope form** of the equation of the line that passes through the point (x_1, y_1) and has a slope of m is

$$y - y_1 = m(x - x_1).$$

EXAMPLE 2 The Point-Slope Form of the Equation of a Line

Find an equation of the line that passes through the point

$$(1, -2)$$

and has a slope of 3.

Solution

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$
$$y - (-2) = 3(x - 1) \quad \text{Substitute for } y_1, m, \text{ and } x_1.$$
$$y + 2 = 3x - 3 \quad \text{Simplify.}$$
$$y = 3x - 5 \quad \text{Solve for } y.$$

The line is shown in Figure 1.5.

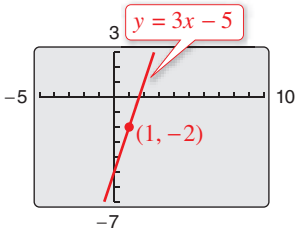

Figure 1.5

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Find an equation of the line that passes through the point $(3, -7)$ and has a slope of 2.

The point-slope form can be used to find an equation of a nonvertical line passing through two points

$$(x_1, y_1) \quad \text{and} \quad (x_2, y_2).$$

First, find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2$$

Then use the point-slope form to obtain the equation

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

This is sometimes called the **two-point form** of the equation of a line.

Remark

When you find an equation of the line that passes through two given points, you need to substitute the coordinates of only one of the points into the point-slope form. It does not matter which point you choose because both points will yield the same result.

EXAMPLE 3 A Linear Model for Profits Prediction

In 2011, Tyson Foods had sales of $32.266 billion, and in 2012, sales were $33.278 billion. Write a linear equation giving the sales y in terms of the year x. Then use the equation to predict the sales for 2013. (Source: Tyson Foods, Inc.)

Solution

Let $x = 0$ represent 2000. In Figure 1.6, let (11, 32.266) and (12, 33.278) be two points on the line representing the sales. The slope of this line is

$$m = \frac{33.278 - 32.266}{12 - 11} = 1.012.$$

Next, use the point-slope form to find the equation of the line.

$$y - 32.266 = 1.012(x - 11)$$

$$y = 1.012x + 21.134$$

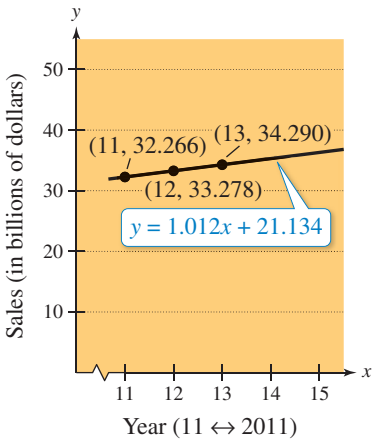

Figure 1.6

Now, using this equation, you can predict the 2013 sales ($x = 13$) to be

$$y = 1.012(13) + 21.134 = 13.156 + 21.134 = \$34.290 \text{ billion}.$$

(In this case, the prediction is quite good—the actual sales in 2013 were $34.374 billion.)

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In 2012, Apple had sales of $156.508 billion, and in 2013, sales were $170.910 billion. Write a linear equation giving the sales y in terms of the year x. Then use the equation to predict the sales for 2014. (Source: Apple, Inc.)

Library of Parent Functions: Linear Function

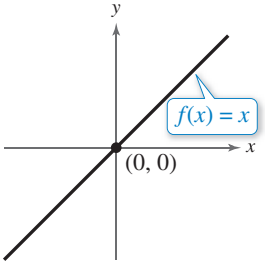
In the next section, you will be introduced to the precise meaning of the term *function*. The simplest type of function is the *parent linear function*

$$f(x) = x.$$

As its name implies, the graph of the parent linear function is a line. The basic characteristics of the parent linear function are summarized below and on the inside cover of this text. (Note that some of the terms below will be defined later in the text.)

Graph of $f(x) = x$

Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(0, 0)$
Increasing



The function $f(x) = x$ is also referred to as the *identity function*. Later in this text, you will learn that the graph of the linear function $f(x) = mx + b$ is a line with slope m and y-intercept $(0, b)$. When $m = 0$, $f(x) = b$ is called a *constant function* and its graph is a horizontal line.

Rob Byron/Fotolia

Sketching Graphs of Lines

Many problems in coordinate geometry can be classified in two categories.

1. Given a graph (or parts of it), find its equation.

2. Given an equation, sketch its graph.

For lines, the first problem can be solved by using the point-slope form. This formula, however, is not particularly useful for solving the second type of problem. The form that is better suited to graphing linear equations is the **slope-intercept form** of the equation of a line, $y = mx + b$.

Slope-Intercept Form of the Equation of a Line

The graph of the equation

$$y = mx + b$$

is a line whose slope is m and whose y-intercept is $(0, b)$.

EXAMPLE 4 Using the Slope-Intercept Form

See LarsonPrecalculus.com for an interactive version of this type of example.

Determine the slope and y-intercept of each linear equation. Then describe its graph.

a. $x + y = 2$ **b.** $y = 2$

Algebraic Solution

a. Begin by writing the equation in slope-intercept form.

$x + y = 2$ Write original equation.

$y = 2 - x$ Subtract x from each side.

$y = -x + 2$ Write in slope-intercept form.

From the slope-intercept form of the equation, the slope is -1 and the y-intercept is

$(0, 2)$.

Because the slope is negative, you know that the graph of the equation is a line that falls one unit for every unit it moves to the right.

b. By writing the equation $y = 2$ in slope-intercept form

$y = (0)x + 2$

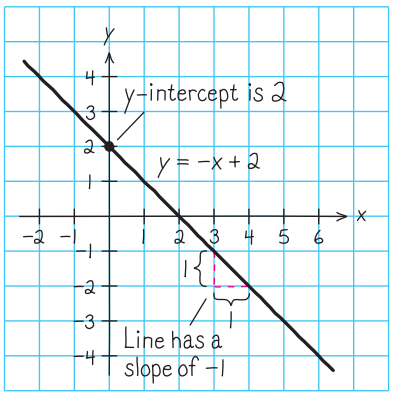
you can see that the slope is 0 and the y-intercept is

$(0, 2)$.

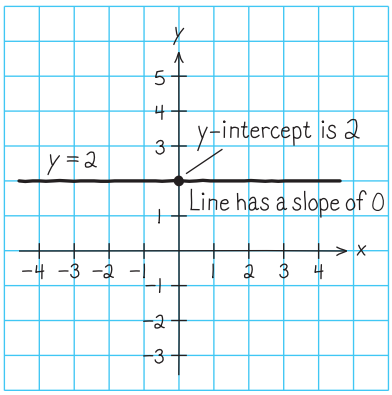
A zero slope implies that the line is horizontal.

Graphical Solution

a.



b.



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Determine the slope and y-intercept of $x - 2y = 4$. Then describe its graph.

From the slope-intercept form of the equation of a line, you can see that a horizontal line ($m = 0$) has an equation of the form $y = b$. This is consistent with the fact that each point on a horizontal line through $(0, b)$ has a y-coordinate of b. Similarly, each point on a vertical line through $(a, 0)$ has an x-coordinate of a. So, a vertical line has an equation of the form $x = a$. This equation cannot be written in slope-intercept form because the slope of a vertical line is undefined. However, every line has an equation that can be written in the **general form**

$$Ax + By + C = 0 \qquad \text{General form of the equation of a line}$$

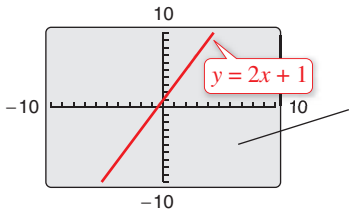
where A and B are not *both* zero.

Summary of Equations of Lines

1.	General form:	$Ax + By + C = 0$
2.	Vertical line:	$x = a$
3.	Horizontal line:	$y = b$
4.	Slope-intercept form:	$y = mx + b$
5.	Point-slope form:	$y - y_1 = m(x - x_1)$

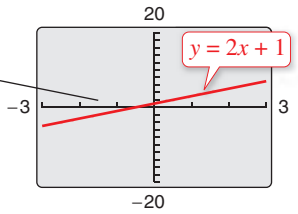
EXAMPLE 5 Different Viewing Windows

When a graphing utility is used to graph a line, it is important to realize that the line may not visually appear to have the slope indicated by its equation. This occurs because of the viewing window used for the graph. For instance, Figure 1.7 shows graphs of $y = 2x + 1$ produced on a graphing utility using three different viewing windows. Notice that the slopes in Figures 1.7(a) and (b) do not visually appear to be equal to 2. When you use a *square setting*, as in Figure 1.7(c), the slope visually appears to be 2.

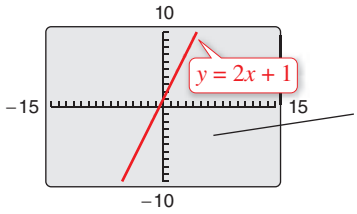


(a) ***Nonsquare setting***

Using a *nonsquare setting*, you do *not* obtain a graph with a true geometric perspective. So, the slope does *not* visually appear to be 2.



(b) ***Nonsquare setting***



Using a *square setting*, you can obtain a graph with a true geometric perspective. So, the slope visually appears to be 2.

(c) ***Square setting***

Figure 1.7

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Use a graphing utility to graph $y = 0.5x - 3$ using each viewing window. Describe the difference in the graphs.

a. Xmin = -5, Xmax = 10, Xscl = 1, Ymin = -1, Ymax = 10, Yscl = 1

b. Xmin = -2, Xmax = 10, Xscl = 1, Ymin = -4, Ymax = 1, Yscl = 1

c. Xmin = -5, Xmax = 10, Xscl = 1, Ymin = -7, Ymax = 3, Yscl = 1

Parallel and Perpendicular Lines

The slope of a line is a convenient tool for determining whether two lines are parallel or perpendicular.

Parallel Lines

Two distinct nonvertical lines are **parallel** if and only if their slopes are equal. That is, $m_1 = m_2$.

Explore the Concept

Graph the lines $y_1 = \frac{1}{2}x + 1$ and $y_2 = -2x + 1$ in the same viewing window. What do you observe?

Graph the lines $y_1 = 2x + 1$, $y_2 = 2x$, and $y_3 = 2x - 1$ in the same viewing window. What do you observe?

EXAMPLE 6 Equations of Parallel Lines

Find the slope-intercept form of the equation of the line that passes through the point $(2, -1)$ and is parallel to the line $2x - 3y = 5$.

Solution

Begin by writing the equation of the line in slope-intercept form.

$$2x - 3y = 5 \qquad \text{Write original equation.}$$

$$-2x + 3y = -5 \qquad \text{Multiply by } -1.$$

$$3y = 2x - 5 \qquad \text{Add } 2x \text{ to each side.}$$

$$y = \frac{2}{3}x - \frac{5}{3} \qquad \text{Write in slope-intercept form.}$$

Therefore, the given line has a slope of

$$m = \tfrac{2}{3}.$$

Any line parallel to the given line must also have a slope of $\frac{2}{3}$. So, the line through $(2, -1)$ has the following equation.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$

$$y - (-1) = \frac{2}{3}(x - 2) \qquad \text{Substitute for } y_1, m, \text{ and } x_1.$$

$$y + 1 = \frac{2}{3}x - \frac{4}{3} \qquad \text{Simplify.}$$

$$y = \frac{2}{3}x - \frac{7}{3} \qquad \text{Write in slope-intercept form.}$$

Notice the similarity between the slope-intercept form of the original equation and the slope-intercept form of the parallel equation. The graphs of both equations are shown in Figure 1.8.

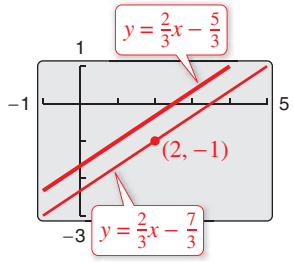


Figure 1.8

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com.

Find the slope-intercept form of the equation of the line that passes through the point $(-4, 1)$ and is parallel to the line $5x - 3y = 8$.

Perpendicular Lines

Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other. That is,

$$m_1 = -\frac{1}{m_2}.$$

EXAMPLE 7 Equations of Perpendicular Lines

Find the slope-intercept form of the equation of the line that passes through the point $(2, -1)$ and is perpendicular to the line $2x - 3y = 5$.

Solution

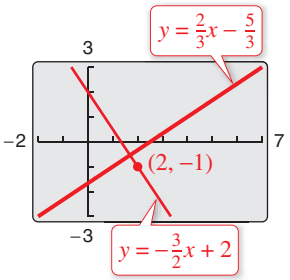
From Example 6, you know that the equation can be written in the slope-intercept form $y = \frac{2}{3}x - \frac{5}{3}$. You can see that the line has a slope of $\frac{2}{3}$. So, any line perpendicular to this line must have a slope of $-\frac{3}{2}$ $\left(\text{because } -\frac{3}{2} \text{ is the negative reciprocal of } \frac{2}{3}\right)$. So, the line through the point $(2, -1)$ has the following equation.

$$y - (-1) = -\tfrac{3}{2}(x - 2) \qquad \text{Write in point-slope form.}$$

$$y + 1 = -\tfrac{3}{2}x + 3 \qquad \text{Simplify.}$$

$$y = -\tfrac{3}{2}x + 2 \qquad \text{Write in slope-intercept form.}$$

The graphs of both equations are shown in the figure.



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Find the slope-intercept form of the equation of the line that passes through the point $(-4, 1)$ and is perpendicular to the line $5x - 3y = 8$.

EXAMPLE 8 Graphs of Perpendicular Lines

Use a graphing utility to graph the lines $y = x + 1$ and $y = -x + 3$ in the same viewing window. The lines are perpendicular (they have slopes of $m_1 = 1$ and $m_2 = -1$). Do they appear to be perpendicular on the display?

Solution

When the viewing window is nonsquare, as in Figure 1.9, the two lines will not appear perpendicular. When, however, the viewing window is square, as in Figure 1.10, the lines will appear perpendicular.

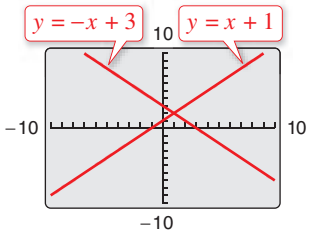


Figure 1.9

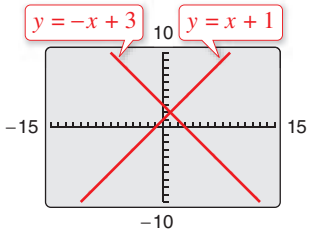


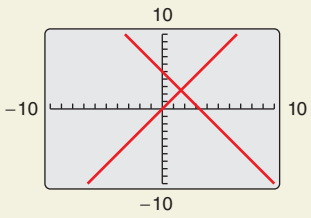
Figure 1.10

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Identify any relationships that exist among the lines $y = 2x$, $y = -2x$, and $y = \frac{1}{2}x$. Then use a graphing utility to graph the three equations in the same viewing window. Adjust the viewing window so that each slope appears visually correct. Use the slopes of the lines to verify your results.

What's Wrong?

You use a graphing utility to graph $y_1 = 1.5x$ and $y_2 = -1.5x + 5$, as shown in the figure. You use the graph to conclude that the lines are perpendicular. What's wrong?



1.1 Exercises

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises.
For instructions on how to use a graphing utility, see Appendix A.

Vocabulary and Concept Check

1. Match each equation with its form.

(a) $Ax + By + C = 0$	(i) vertical line
(b) $x = a$	(ii) slope-intercept form
(c) $y = b$	(iii) general form
(d) $y = mx + b$	(iv) point-slope form
(e) $y - y_1 = m(x - x_1)$	(v) horizontal line

In Exercises 2 and 3, fill in the blank.

2. For a line, the ratio of the change in y to the change in x is called the _____ of the line.

3. Two lines are _____ if and only if their slopes are equal.

4. What is the relationship between two lines whose slopes are -3 and $\frac{1}{3}$?

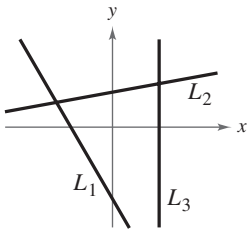
5. What is the slope of a line that is perpendicular to the line represented by $x = 3$?

6. Give the coordinates of a point on the line whose equation in point-slope form is $y - (-1) = \frac{1}{4}(x - 8)$.

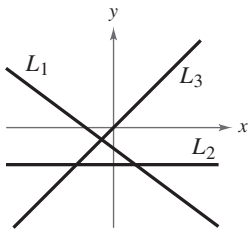
Procedures and Problem Solving

Using Slope **In Exercises 7 and 8, identify the line that has the indicated slope.**

7. (a) $m = \frac{2}{3}$ (b) m is undefined. (c) $m = -2$

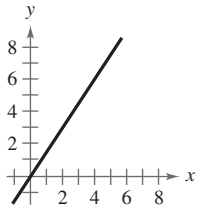


8. (a) $m = 0$ (b) $m = -\frac{3}{4}$ (c) $m = 1$

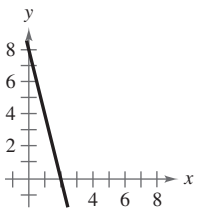


Estimating Slope **In Exercises 9 and 10, estimate the slope of the line.**

9.



10.



Sketching Lines **In Exercises 11 and 12, sketch the lines through the point with the indicated slopes on the same set of coordinate axes.**

	Point	*Slopes*			
11.	$(2, 3)$	(a) 0	(b) 1	(c) 2	(d) -3
12.	$(-4, 1)$	(a) 4	(b) -2	(c) $\frac{1}{2}$	(d) Undefined

Finding the Slope of a Line **In Exercises 13–16, find the slope of the line passing through the pair of points. Then use a graphing utility to plot the points and use the *draw* feature to graph the line segment connecting the two points. (Use a *square setting*.)**

13. $(0, -10), (-4, 0)$ **14.** $(2, 4), (4, -4)$

15. $(-6, -1), (-6, 4)$ **16.** $(4, 9), (6, 12)$

Using Slope **In Exercises 17–24, use the point on the line and the slope of the line to find three additional points through which the line passes. (There are many correct answers.)**

Point	*Slope*
17. $(2, 1)$	$m = 0$
18. $(3, -2)$	$m = 0$
19. $(1, 5)$	m is undefined.
20. $(-4, 1)$	m is undefined.
21. $(0, -9)$	$m = -2$
22. $(-5, 4)$	$m = 4$
23. $(7, -2)$	$m = \frac{1}{2}$
24. $(-1, -6)$	$m = -\frac{1}{3}$